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A METHOD FOR THE DETERMINATION OF LOCAL CONVECTIVE HEAT TRANSFER FROM A CYLINDER PLACED NORMAL TO AN AIR STREAM

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Abstract—The method described in this paper is based on the solution of the equation for the steady temperature distribution in the wall of the tube, using as boundary conditions the measured temperature distribution on the outer surface and a known, constant heat-transfer coefficient at the inner tube surface.

Results are given of measurements for the case of an undisturbed incident flow.

NOMENCLATURE

- U, air flow velocity, m/s;
- t, temperature, degC;
- t_0, temperature of liquid, degC;
- t_1, temperature of inner wall, degC;
- t_2, temperature of outer wall, degC;
- t_3, temperature of gas, degC;
- y, dimensionless temperature ratio, $(t - t_3)/(t_0 - t_3)$;
- r, radial co-ordinate, m;
- r_1, inside radius of tube, m;
- r_2, outside radius of tube, m;
- x, dimensionless radial co-ordinate, r/r_2;
- a_n, Fourier coefficient;
- φ, polar co-ordinate angle;
- α_1, heat-transfer coefficient at inner surface, kcal/m^2 h degC;
- α_2, heat transfer coefficient at outer surface, kcal/m^2 h degC;
- λ_0, thermal conductivity of liquid, kcal/m h degC;
- λ_1, thermal conductivity of tube, kcal/m h degC;
- λ_2, thermal conductivity of gas, kcal/m h degC;
- β_1, $2\lambda_1/\lambda_0$;
- β_2, $2\lambda_1/\lambda_2$;
- γ, Nu_1/β_1;
- Nu, average Nusselt number at outer surface;
- Nu_1, Nusselt number at inner wall;
- Nu_2, Nusselt number at outer wall;
- Nu_s, Nusselt number at stagnation point.

INTRODUCTION

In connexion with fundamental research at this institute on the influence of turbulence on the transfer of heat from a circular cylinder placed normal to an air stream, it became necessary to measure local heat-transfer rates. Various methods for doing this are indicated in the literature [1–5]. A careful study, however, reveals that against each of these methods objections can be raised, especially with regard to the requirement that the measuring system should introduce only negligible modifications of the thermal condition of the tube wall and into the flow field around the tube.

In an attempt to avoid the various difficulties engaged with known techniques the method described in this paper was devised.

An essential feature is that only the temperature distribution on the outer surface of the tube is measured. This can be performed by means of an unheated platinum-film resistance of extremely small thickness and such small cross-dimensions that its influence on the temperature field in the tube wall and on the air flow around the cylinder is negligible.

FUNDAMENTAL CONSIDERATIONS

Only the steady state is considered. A cylindric tube is placed in a flowing medium, say air. The incident air flow is assumed to have a uniform velocity and temperature (Fig. 1)

A hot liquid is pumped through the tube and

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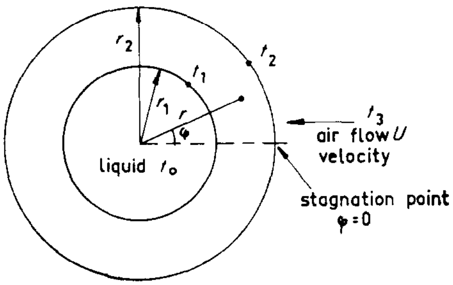

FIG. 1. Geometry of tube and symbols used in the mathematical treatment.

heat is transferred from this liquid across the wall of the tube to the air. If the rate of flow of the liquid is sufficiently high, the axial flow of heat in the tube wall can be made very small in comparison with the radial and tangential flow of heat. In this case the temperature field in the tube wall will be essentially two-dimensional. Introducing dimensional variables,

$$y = (t - t_3)/(t_0 - t_3) \quad \text{and} \quad x = r/r_2,$$

the equation for this field is:

$$\frac{\partial^2 y}{\partial x^2} + \frac{1}{x}\frac{\partial y}{\partial x} + \frac{1}{x^2}\frac{\partial^2 y}{\partial \varphi^2} = 0. \tag{1}$$

Note that the tangential flow of heat is preserved in this equation.

As boundary conditions, the following are chosen:

(a) A given temperature distribution at the outer surface.
(b) A given heat-transfer coefficient, independent of x and φ, at the inner surface.

Experimentally it is proved [1, 2] that symmetry exists with respect to $\varphi = 0$. This simplifies the analysis. The first boundary condition (a) can now be formulated as:

$$y_2 = \sum_0^\infty a_n \cos n\varphi \quad \text{for } x = 1. \tag{2}$$

The coefficients a_n are found by a Fourier analysis of the measured distribution y_2. The boundary condition (b) is put in a mathematical form, using the definition of heat-transfer coefficient and equating the heat flow from liquid to wall to the radial heat flow in the tube wall at the inner surface:

$$Nu_1(1 - y_1) = -\beta_1 x_1 \left(\frac{\partial y}{\partial x}\right)_{x=x_1}. \tag{3}$$

Here $Nu_1 = (2\alpha_1 r_1)/\lambda_0$ and $\beta_1 = (2\lambda_1)/\lambda_0$.

Now the solution is written as a Fourier series:

$$y = \sum_0^\infty f_n(x) \cos n\varphi. \tag{4}$$

The functions $f_n(x)$ are found by substitution of (4) into (1). This leads to:

$$y = A_0 + B_0 \ln x + \sum_1^\infty (A_n x^n + B_n x^{-n}) \cos n\varphi. \tag{5}$$

Now the boundary conditions are used to express the coefficients A_n and B_n as functions of the coefficients a_n occurring in equation (2). The result of the calculations, the details of which we omit here, is expressible in the form:

$$\left.\begin{aligned} y = 1 &+ \frac{a_0 - 1}{(1/\gamma) - \ln x_1}\left(\frac{1}{\gamma} + \ln\frac{x}{x_1}\right) \\ &+ \sum_1^\infty a_n \left\{x_1^{-n} - \frac{\gamma - n}{\gamma + n} x_1^{n}\right\}^{-1} \\ &\times \left\{\left(\frac{x}{x_1}\right)^n - \frac{\gamma - n}{\gamma + n}\left(\frac{x}{x_1}\right)^{-n}\right\} \cos n\varphi \end{aligned}\right\} \tag{6}$$

in which for simplification the dimensionless parameter $\gamma = Nu_1/\beta_1$ is introduced. From this distribution of the temperature in the wall of the tube, the local Nu number at the outer tube surface is derived by an equation that is analogous to equation (3):

$$Nu_2 = -\frac{\beta_2}{y_2}\left(\frac{\partial y}{\partial x}\right)_{x=1} \tag{7}$$

in which $Nu_2 = (2\alpha_2 r_2)/\lambda_2$ and $\beta_2 = 2\lambda_1/\lambda_2$. The final result, and the basis of the method described, is given by:

$$Nu_2 = \frac{-\beta_2}{y_2}\left[\frac{a_0 - 1}{(1/\gamma) - \ln x_1} + \sum_1^\infty \left\{ n\, a_n \frac{x_1^{-2n} + \dfrac{(\gamma - n)}{(\gamma + n)}}{x_1^{-2n} - \dfrac{(\gamma - n)}{(\gamma + n)}} \cos n\varphi \right\}\right]. \tag{8}$$

Apart from the coefficients a_n, determined by the distribution y_2, as parameters in equation (8), only x_1, β_2 and γ occur. x_1 is easily deduced from the geometry of the tube and is a constant, $\beta_2 = 2\lambda_1/\lambda_2$ requires only the knowledge of thermal conductivities. In $\gamma = Nu_1/\beta_1$, however, the inside heat-transfer number Nu_1 occurs. There are several ways to determine its value. One could calculate it from known correlations. Another way could be to make an experimental evaluation of a heat balance along the tube. According to our experience, however, a very suitable way is as follows. In the study of the influence of flow conditions on the local Nu numbers for a given set of experiments, one can keep constant the temperatures of the air and of the liquid, as well as the velocity of the liquid. In this case β_1 and γ will be constants and independent of the air-flow conditions. Now for the case of an undisturbed incident gas flow, the local Nu at the stagnation point is given by Squire [6]:

$$Nu_s = 1{\cdot}14\, Re^{0{\cdot}5}\, Pr^{0{\cdot}371}. \qquad (9)$$

This equation proved exceedingly well verified by measurements of Giedt [1] and of Schmidt and Wenner [2]. Therefore γ can be found by determination of Nu_s from equation (8) under conditions of undisturbed incident flow, in such a way that it equals the value obtained from equation (9). This can be done in a simple though accurate way by observing that because of symmetry there is no tangential heat flow at the stagnation point, so that as a good approximation:

$$Nu_s = \frac{-\beta_2}{y_s}\,\frac{y_s - 1}{(1/\gamma) - \ln x_1}. \qquad (10)$$

Now y_s being measured, x_1 and β_2 being known, γ is obtained immediately from equations (9) and (10). The actual performance of this procedure is discussed elsewhere in this paper.

Concerning the numerical Fourier analysis and synthesis it can be remarked that easy methods for it are readily available. In this paper, twelve harmonics are taken into consideration. Strictly speaking the surface temperature need be measured in this case only at $\varphi = k \times 15°$ $(k = 0, 1, 2, \ldots 12)$.

EXPERIMENTAL DETAILS

A. *The test tube*

A precision-bore Pyrex tube is used; $r_1 = 10{\cdot}02$ mm, $r_2 = 11{\cdot}60$ mm, and the length is 160 mm. The thermal conductivity of the glass at 60°C is 1·07 kcal/m h degC.

Instead of pumping the liquid through the tube in a straightforward manner, an arrangement is chosen in which the liquid flows through a 1-mm wide annulus formed between the inner wall of the Pyrex tube and the outer wall of a concentric inner brass tube having an outside diameter of 18·04 mm (Fig. 2). The inner tube is

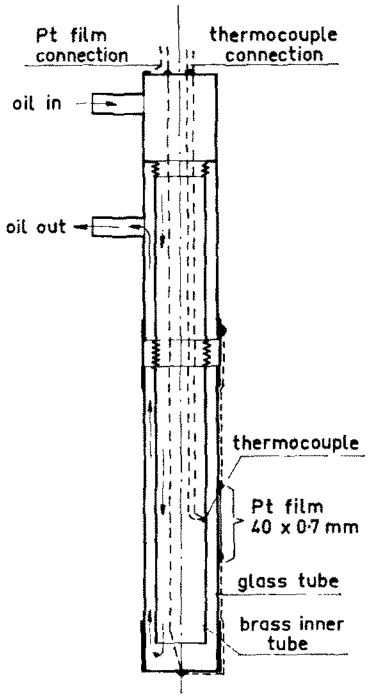

FIG. 2. Diagram of test tube (actual dimensions are given in the text).

kept accurately centric by means of small spacers near the top and the bottom of the tube. The liquid flows downwards through the brass tube and upwards through the annulus. Metal–glass connexions are made by means of Araldite. Bellows are introduced to obtain some flexibility to facilitate the assembling of the unit.

The test tube is mounted vertically in the test section of an open-circuit type wind tunnel, having a square cross-section of 250 × 250 mm. The liquid pumped through the tube is Shell oil P39. The oil is pumped at a constant rate of 10 l/min by means of a gear pump from a thermostat containing 40 l of it, through thermally insulated tubing to the test tube and back to the thermostat. In the experiments described here the oil temperature in the test tube could be fixed in this way at 78·8 ± 0·1 °C, independent of air-flow conditions.

B. *Temperature measurements*

Opposite to the place where the surface temperature of the glass tube is measured, in the brass inner tube a copper–constantan thermocouple is embedded in Araldite filling a small hole in the wall. The junction of this couple is made flush with the outer surface of this inner tube by means of polishing. Calibration of the couple using a precision liquid thermostat enables the liquid temperature to be measured with an accuracy of 0·02 degC. The e.m.f. of this couple is measured by means of a Diesselhorst compensator.

The surface temperature of the Pyrex tube is measured along a generating line of the cylinder by means of a platinum-film resistance of 40 mm length, 0·7 mm width and of 5×10^{-5} mm thickness. The film resistance is produced by firing a solution called Liquid Bright Platinum 05-X (Hanovia) on to the tube. Connexions are made by means of fired silver paint Du Pont

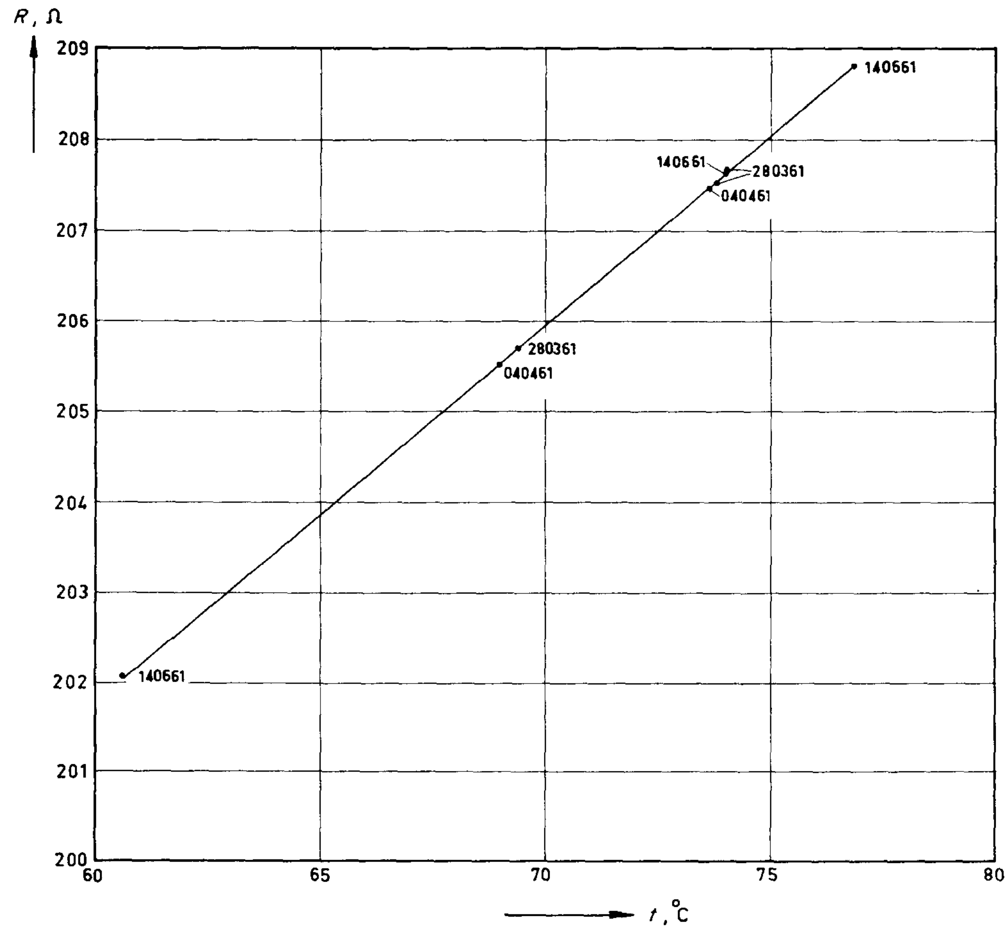

FIG. 3. Calibrations for a Pt-film resistance at various dates (e.g. 280361 = March 28th, 1961).

6704. Silver wires can be soldered to these connexions as conductors. These conductors, as well as those of the thermocouple mentioned above, run through the oil to the upper side of the test tube.

The platinum resistance obtained in this way shows a perfectly linear relationship between resistance and temperature, and has an excellent long-term stability. Fig. 3 shows results of calibrations covering a period of about 3 months. The resistance is measured by means of a precision Wheatstone bridge, the current through the film being 50 μA. Calibration in a liquid thermostat showed that the accuracy of the surface-temperature measurement is about 0·1 degC.

The air temperature is measured upstream of the test tube by means of a mercury-in-glass thermometer calibrated to 0·1 degC.

RESULTS OF MEASUREMENTS

In the empty wind tunnel, the intensity of the free stream turbulence is about 0·2 per cent. The results given here pertain to this condition.

The air velocity is constant across the region of interest for the heat-transfer measurements.

From the thermal conductivities of the tube wall at the average temperature of 60°C and of the air at the average film temperature of 40°C it followed that $\beta_2 = 91{\cdot}85$. For the tube we used $x_1 = (10{\cdot}02/11{\cdot}60) = 0{\cdot}879$, γ is determined following the procedure described earlier. Fig. 4 shows a plot of $(1 - y_s)/y_s$, obtained from measurements of y_s, against $U^{1/2}$. The proportionality following from equations (9) and (10) is confirmed with great precision. At the same time this proves γ to be independent of U, as it ought to be. The value found from the plot for γ is 8·34.

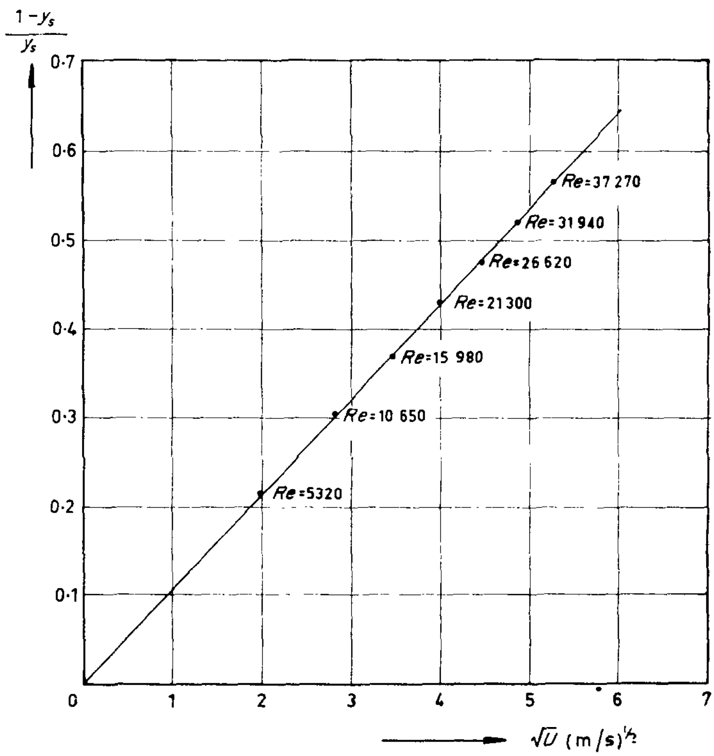

FIG. 4. Relationship between stagnation-point heat transfer and air velocity.

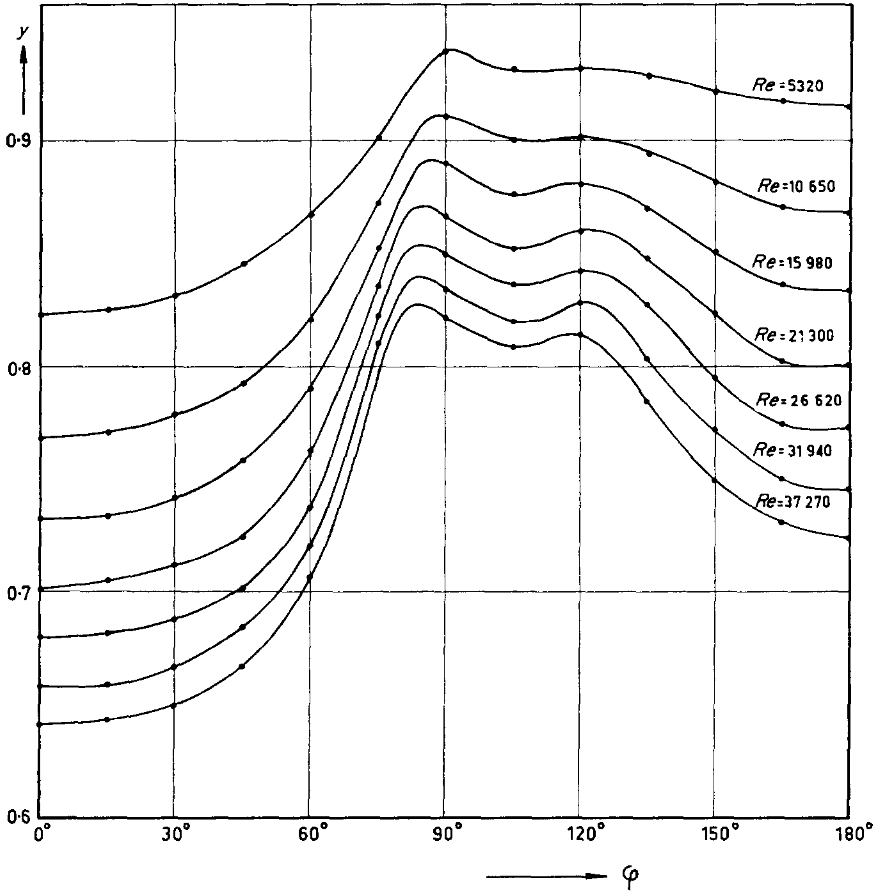

FIG. 5. Dimensionless temperature distributions on the outer surface of the test tube.

Except for the coefficients a_n all parameters needed are now known.

Figure 5 gives measurements of surface temperatures obtained by rotation of the test tube through angles of 15°. From repeated measurements not shown here, the reproducibility of y is evaluated at about 0·3 per cent. Not shown either are points determining the location of the maximum value of y. Perfect symmetry was shown to exist between the two halves of the profiles, so the experiments were restricted to φ between 0° and 180°.

The results of the calculations of local values of the Nusselt number are given in Fig. 6. A more detailed study of these curves will be given in a future paper concerning the influence of turbulence on the heat transfer of a cylinder. As the main purpose of this paper is the description of the method, we restrict ourselves at this place to the following remarks.

The average value of Nu is easily obtained from the graphs, by integration. From a collection of all known data, Van der Hegge Zijnen [7] derived the following equation for the

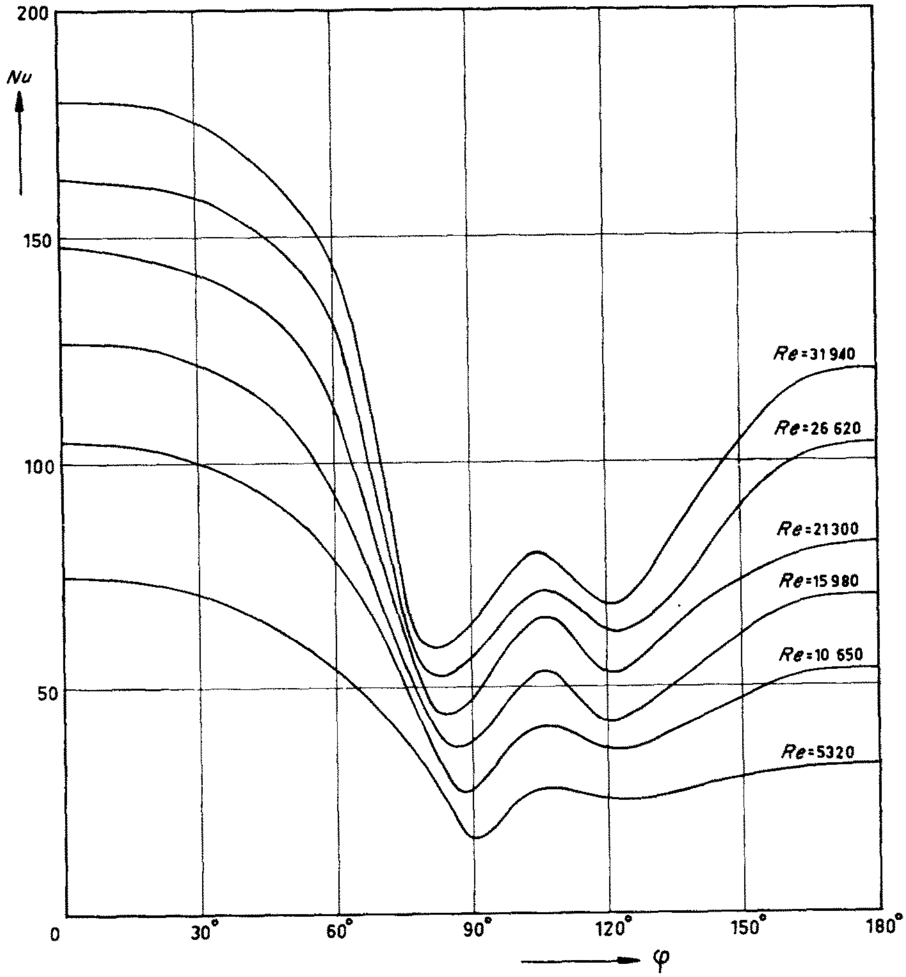

FIG. 6. Distribution of *Nu* on the outer cylinder surface.

average *Nu* of a cylinder in cross flow of air:

$$Nu = 0{\cdot}35 + 0{\cdot}5\, Re^{0{\cdot}5} + 0{\cdot}001\, Re, \quad (11)$$

thermal properties being evaluated at the film temperature.

This function is plotted, together with the average *Nu* obtained from our measurements, in Fig. 7. There is good agreement. The difference in slope of the curves is probably to be attributed to the effect of turbulence. On the average, the data from literature on which Van der Hegge Zijnen based his correlation were probably obtained for higher intensities than 0·2 per cent. Preliminary measurements showed us an increasing slope with increasing intensity of turbulence.

In the same graph the average *Nu* numbers are given for the regions 0°–90° and 90°–180°.

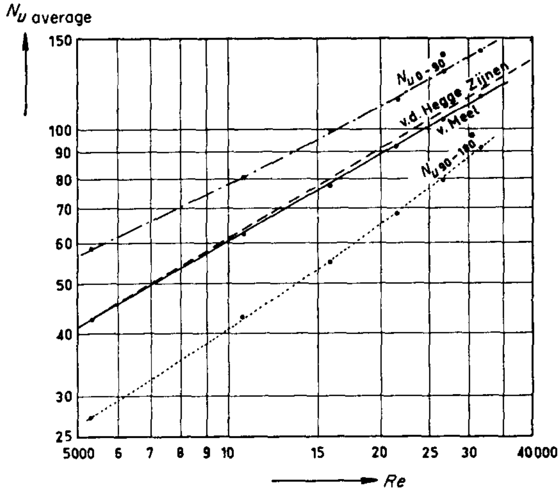

FIG. 7. Average Nu on the outer cylinder surface related to Re.

Perfectly straight lines are not obtained on the logarithmic diagram. For this range of Re numbers, however, $Nu_{0^\circ-90^\circ}$ is, on the average, proportional to $Re^{0 \cdot 50}$ and $Nu_{90^\circ-180^\circ}$ to $Re^{0 \cdot 675}$.

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Résumé—La méthode décrite dans cet article est fondée sur la solution de l'équation relative à une distribution de température constante à la paroi du tube, en prenant pour conditions auxl imites la distribution de température mesurée sur la face extérieure et un coefficient de transmission de chaleur constant, connu, sur la paroi interne du tube.

Des résultats expérimentaux sont donnés dans le cas d'un écoulement incident non perturbé.

Zusammenfassung—Die hier beschriebene Methode beruht auf der Lösung der Gleichung für stetige Temperaturverteilung in der Rohrwand. Mit Hilfe der Grenzbedingungen, der gemessenen Temperaturverteilung an der Rohraussenfläche und einem bekannten, konstanten Wärmeübergangskoeffizienten an der Rohrinnenwand erhält man diese Lösung.

Für ungestörte Anströmung sind die Messergebnisse angegeben.

Аннотация—Метод, изложенный в данной статье; основан на решении уравнения стационарной теплопроводности в стенке полого цилиндра, когда в качестве граничных условий используются экспериментально замеренное распределение температуры на внешней поверхности цилиндра и известный постоянный коэффициент теплообмена на внутренней поверхности цилиндра.

Приводятся результаты определения коэффициента теплообмена для случая невозмущённого внешнего течения.